

quo resultabit quantitas, in quam solùm ingredientur  $x$ , Fig. &  $dx$ , quæ integrata dabit aream quæsitam, addendo (quod semper præ oculis habere debet) constantem  $C$ . Patet enim, quod spatium infinitesimum  $PMmp$  non solum est differentiale quantitatis  $AMP$ , sed etiam hujus  $CLPM$ ; consequenter integrale quantitatis  $PMmp$  denotabit utramque aream: ergo necesse erit addere integrali invento unam constantem, indicantem differentiam inter aream quæsitam, & eam quæ ex calculo resultat, supponendo semper quod ang. coordinatarum est rectus.

841 Prob. 1. *Invenire aream triang. cujuslibet*  $ABC$ . 108

Sol. Sit basis  $AB = b$ , altitudo  $DC = a$ ; ac ducendo lin.  $KG$  parallelam ad basim, sit  $CF = x$ ,  $KG = y$ ; ob rationem basium parallelarum, erit  $b : a :: y : x$ ; ergo  $ay = bx$ ,  $y = \frac{bx}{a}$ ,  $ydx = \frac{bx dx}{a}$ , ac consequenter  $\int ydx = \int \frac{bx dx}{a} = \frac{bx^2}{2a}$ ; quæ expressio repræsentat quamlibet portionem  $CKG$  areæ triang.  $ABC$ ; ergo si  $x$  ob suum incrementum fit  $= a$ , erit  $\frac{bx^2}{2a} = \frac{ba^2}{2a} = \frac{1}{2} ba$ , & habebitur area totius triang. conformiter ad dicta in elem. Geomet. ( 493 ).

842 Prob. 2. *Invenire aream parabolæ.*

Sol. Cum sit æquatio ad hanc curvam  $y^2 = px$ , erit  $y = p^{\frac{1}{2}} x^{\frac{1}{2}}$ ,  $ydx = p^{\frac{1}{2}} x^{\frac{1}{2}} dx$ ; ergo  $\int ydx = \int p^{\frac{1}{2}} x^{\frac{1}{2}} dx = \frac{p^{\frac{1}{2}} x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} p^{\frac{1}{2}} x^{\frac{3}{2}}$   $= \frac{2}{3} xy$ ; hoc est, *area parabolæ est æqualis duobus tertiis rectanguli abscissæ per ordinatam.*

843 Prob. 3. *Invenire aream circuli.*

Sol. Hujus æquatio, numerando abscissas è centro, & 99

Fig. dato quod ejus radius sit  $= a$ , est  $y^2 = a^2 - x^2$ ; ergo  
 $y = \sqrt{a^2 - x^2}$ ,  $ydx = dx \sqrt{a^2 - x^2}$ ;  $S. ydx =$   
 $S. dx \sqrt{a^2 - x^2} = ax - \frac{x^2}{2.3a} - \frac{1 x^5}{2.4.5a^3} - \frac{1.3x^7}{2.4.6.7a^5} - \&c.$   
 quæ expressio representat aream cujuslibet spatii circu-  
 laris  $CPNG$ , cum sit  $CP = x$ . Quare si consideramus  
 quod fit  $x = a$ , & hic valor substituitur in expressione  
 precedente, productum  $a^2 - \frac{a^2}{2.3} - \frac{1.3a^2}{2.4.5} - \frac{1.3.5a^2}{2.4.6.7} - \&c.$   
 representabit aream quadrantis  $CGNA$ , cujus quadruplum  
 erit area circuli integri, aut tota ejus quadratura.

844 Prob. 4. *Invenire aream elipsis.*

Sol. In hac curva numerando abscissas è centro, cum sit  
 $CP = x$ , semiaxis minor  $= b$ , & major  $= a$ , est  $y =$   
 $\frac{b}{a} \sqrt{a^2 - x^2}$ ; ergo  $ydx = \frac{bdx}{a} \sqrt{a^2 - x^2}$ ,  $S. ydx =$   
 $S. \frac{bdx}{a} \sqrt{a^2 - x^2} =$  portioni spatii elliptici cujuslibet  
 $CPMD$ , & cum sit (843)  $S. dx \sqrt{a^2 - x^2} =$  spatio  
 circulari  $CGNP$ ; patet quod erit  $CPMD : CGNP :: \frac{b}{a} : 1$   
 $:: b : a$ , hoc est, *spatium ellipticum inter duas ordinatas con-*  
*tentum, est ad spatium correspondens circuli circumscripti*  
*ad elipsim, sicut axis minor elipsis ad suum axim majo-*  
*rem; ac consequenter tota elipsis habet cum toto circulo*  
*eandem rationem.*

## ARTICULUS VI.

### *De Rectificatione curvarum.*

845 **T**Heor. Formula generalis exprimens differen-  
 tiale, seu elementum infinitesimum arcus  
 cujuslibet curvæ est  $\sqrt{dx^2 + dy^2}$ .

Vidimus ( 802 ) quod arcus  $Mm$  est elementum *Fig.*  
 infinitesimum arcus  $AM$ ; sed ille arcus ob suam exigui- 105  
 tatem necessariò confundi debet cum portione  $Mm$   
*tang.*  $TM$ , ac proinde erit hypotenusa *triang.*  $MRm$   
 rectanguli in  $R$ , datis  $Mm = \sqrt{dx^2 + dy^2}$ ; ergo &c.

846 Coroll. Cum sit arcus infinitesimus  $Mm = \sqrt{dx^2 + dy^2}$  differentiale arcus  $AM$ , necessario erit  $S.\sqrt{dx^2 + dy^2} = AM$ . Sic ad rectificandam quamlibet curvam, differentiabitur ejus æquatio, deducetur valor ex  $dx^2$ , seu  $dy^2$ , qui substituetur in formula generali, ac postea integrando habebitur curva ad rectam reducta.

847 Prob. 1. Rectificare arcum quemlibet  $DM$  parabolaë.

Sol. Æquatio ad hanc curvam, cum sit parameter 97  
 $= p$ , &  $AP = x$ , est  $y^2 = px$ , quæ differentiatâ dat  
 $2ydy = p dx$ ; ergo  $dx = \frac{2ydy}{p}$ ,  $dx^2 = \frac{4y^2 dy^2}{p^2}$ , cujus valor  
 substitutus in formula generali dat  $\sqrt{dx^2 + dy^2} =$   
 $\sqrt{\frac{4y^2 dy^2}{p^2} + dy^2} = \frac{dy}{p} \sqrt{p^2 + 4y^2}$ ; ac consequenter  
 $S.\sqrt{dx^2 + dy^2} = S.\frac{dy}{p} \sqrt{p^2 + 4y^2} = y + \frac{2y^3}{3p^2} - \frac{2y}{5p^4} +$   
 $\frac{4y^7}{7p^6} - \&c.$  expressio significans quemlibet arcum  $AM$  pa-  
 rabolaë jam rectificatum.

848 Prob. 2. Rectificare quemlibet arcum  $GM$  circuli.

Sol. Numerando abscissas è centro, & cum sit radius  $= a$  95  
 est in hac curva  $y = \sqrt{a^2 - x^2}$ ; ergo  $dy = \frac{-x dx}{\sqrt{a^2 - x^2}}$ ,  
 $dy^2 = \frac{x^2 dx^2}{a^2 - x^2}$ ; cujus valor substitutus in formula generali  
 dat  $\sqrt{dx^2 + dy^2} = \frac{adx}{\sqrt{a^2 - x^2}}$ , ac consequenter . . .

Fig.  $S. \sqrt{dx^2 + dy^2} = S. \frac{adx}{\sqrt{a^2 - x^2}} = x + \frac{x^3}{2.3a^2} + \frac{1.3.5x^5}{2.4.6.7a^6} + \&c.$

in qua expressione repræsentatur valor arcus cujuslibet circularis  $GM$  rectificati, & faciendo in ea  $x = a$  habebitur rectificatio quadrantis  $GMA$ , cujus quadruplum exprimet rectificationem totius peripheriæ circularis.

## ARTICULUS VII.

*De Cubicatione, seu mensura solidorum.*

849 **T**Heor. *Formula generalis exprimens differentiale, aut elementum infinitesimum unius solidi, producti per revolutionem curvæ circum suum axim, est  $\frac{py^2 dx}{2r}$  repræsentando  $y$  ordinatas,  $x$  abscissas, &  $r$ : p rationem radii ad peripheriam.*

105 Considerando quod curva  $AM$  gyrat circum suum axim  $AQ$ , absdubio generabit solidum, in quo quælibet sectio perpendicularis ad axim erit circulus habens pro radio ordinatam  $y$  curvæ; consequenter circumferentia singulis his sectionibus correspondens erit  $= \frac{py}{r}$  (499), & ejus area  $= \frac{py^2}{2x}$  (496).

Hoc supposito, si consideremus omne solidum revolutionis conflatum ex sectionibus infinite parvis, & ad axim perpendicularibus, differentia inter duas has superficies planas cujuslibet nulla erit, cum sit infinite parva; quare contemplari singulæ poterunt tanquam cylinder, cujus basis est superficies circuli descripti radio  $y$ , & altitudo differentiale  $dx$ ; ergo cujuslibet soliditas infinitesima  $= \frac{py^2 dx}{2r}$  (600).

Cum

850 Cum sit itaque  $\frac{py^2 dx}{2r}$  elementum infinitesimum *Fig.* solidorum revolutionis, habebimus quod  $S. \frac{py^2 dx}{2r}$  exprimet omne solidum. Sic ut illa formula ad casus speciales contrahatur, deducetur ex æquatione plani generatoris valor ex  $y^2$  in expressionibus ex  $x$ : substituetur in formula generali, ac postmodum integrando, resultabit valor, aut mensura soliditatis totius solidi revolutionis.

851 Schol. Licet solida non sint revolutionis, possunt considerari tanquam composita ex fragmentulis, aut sectionibus infinitè parvis, & inter se parallelis. Quare appellando  $t$  unam è superficibus cujuslibet sectionis, &  $x$  quamlibet portionem lineæ ad eas perpendicularis, erit soliditas singularum  $= tdx$ ; ac consequenter  $S.tdx$  exprimet soliditatem totius solidi; & ita ad hoc ut hæc forma applicabilis reddatur, nil superest nisi ut in casibus specialibus inveniatur valor ex  $t$  in expressionibus ex  $x$ .

Ex. Proponatur invenienda hoc medio soliditas py- 81  
ramidis  $OGHK$ ; ad hoc supponemus ejus basim  $GHK$   
 $= c^2$ , altitudinem  $DO = a$ , & appellando  $x$  portionem  
 $dO$ , distantiam cujuslibet sectionis, cujus superficies est  
 $gbk = t$ , ad verticem  $O$ , habebimus (544)  $c^2 : t :: a^2 :$   
 $x^2$ ; ergo  $t = \frac{c^2 x^2}{a^2}$ ,  $tdx = \frac{c^2 x^2 dx}{a^2}$ ;  $S.tdx = S. \frac{c^2 x^2 dx}{a^2} = \frac{c^2 x^3}{3a^2}$ ,  
quæ quidem expressio repræsentat soliditatem unius por-  
tionis pyramidalis cujuslibet  $Ogbk$ ; & si supponimus  $x = a$ , 81  
expressio  $\frac{c^2 x^3}{3a^2}$  convertetur in hanc  $\frac{1}{3} c^2 a$ , in qua repræ-  
sentatur soliditas totius pyramidis  $OGHK$ , sicut in princi-  
piis Geometriæ (606),  $= \frac{1}{3} c^2 a = \frac{1}{3} c^2 a$ .

852 Prob. I. Invenire soliditatem coni  $GBL$ , gene- 76  
ra-

Fig. rati per rotationem triang. rectanguli SCL circum cathetum CS.

Sol. Sit  $CS = b$ ,  $CL = a$ , & ducendo lin.  $cl$  parallelam ad  $CL$ , sit  $cS = x$ , &  $cl = y$ , habebimus  $b : a :: x : y$ ; consequenter  $ax = by$ , æquatio plani generatoris conicæ, & ejus virtute habebimus  $y = \frac{ax}{b}$ ,  $y^2 = \frac{a^2 x^2}{b^2}$ , cujus valor substitutus in formula generali solidi infenitesimi dat  $\frac{py^2 dx}{2r} = \frac{pa^2 x^2 dx}{2rb^2}$ ,  $\int \frac{py^2 dx}{2r} = \int \frac{pa^2 x^2 dx}{2rb^2} = \frac{pa^2 x^3}{3 \cdot 2 \cdot rb^2}$ , cujus expressio repræsentat soliditatem unius portionis conicæ cujuslibet  $bIS$ ; & siquidem supponimus  $x = b$ , convertetur in  $\frac{pa^2 b^3}{3 \cdot 2r} = \frac{pa^2}{2r} \cdot \frac{1}{3} b^3$  in qua repræsentatur soliditas totius conicæ  $SBL$  (§. 607).

853 Prob. 2. Invenire soliditatem conoidis parabolici.

Sol. Vocatur generatim conois omne solidum formatum per revolutionem cujuslibet è tribus sectionibus conicis circum suum axim, suam applicatam, aut suam tangentem.

Et speciatim, si sectio generatrix est semiparabola, solidum resultans dicitur *conois parabolica*, aut *paraboloides*: si semihyperbola, *conois hyperbolica*, aut *hyperboloides*: si semielipsis, *conois elliptica*, aut *elipsoides*; tamen si semielipsis gyrat circa suum axim majorem, solidum dicitur *elipsoides prolongata*; si autem circum axim minorem, *contracta*.

Hoc supposito, vocando  $a$  parametrum parabolæ, erit ejus æquatio  $y^2 = ax$ , cujus valor substitutus in formula generali solidi differentialis dat  $\frac{py^2 dx}{2r} = \frac{pax dx}{2r}$ , ac consequenter  $\int \frac{py^2 dx}{2r} = \int \frac{pax dx}{2r} = \frac{pax^2}{2 \cdot 2r} = \frac{pax}{2r} \cdot \frac{1}{2} x = \frac{py^2}{2r} \cdot \frac{1}{2} x$ , expressio manifestans soliditatem cujuslibet conoidis para-

bolicæ esse æqualem producto suæ basis  $\frac{py^2}{2x}$  per  $\frac{1}{2}x$  di-*Fig.*  
midium suæ altitudinis.

854 Coroll. Ergo cylinder, paraboloides, & conus æqualium basium & altitudinum sunt inter se sicut  $x$ ,  $\frac{1}{2}x$ ,  $\frac{1}{3}x$ , hoc est, sicut 1,  $\frac{1}{2}$ ,  $\frac{1}{3}$ , seu sicut 6, 3, 2.

855 Prob. 3. *Invenire soliditatem elipsoides prolongatæ.*

Sol. Supponendo quod axis major elipsis est  $= 2a$ , & minor  $= 2b$ , habebimus  $y^2 = \frac{b^2}{a^2}(2ax - x^2)$ , cujus valor substitutus in formula generali dat  $\frac{py^2 dx}{2x} = \frac{pb^2}{2xa^2}(2axdx - x^2 dx)$ ; ac consequenter  $S. \frac{py^2 dx}{2r} = S. \frac{pb^2}{2ra^2}(2axdx - x^2 dx) = \frac{pb^2}{2ra^2}(ax^2 - \frac{1}{3}x^3)$  quæ expressio significat soliditatem cujuslibet portionis elipsoidicæ; & supponendo quod deveniat  $x = 2a$ , illa expressio convertetur in  $\frac{2pb^2 a}{3r} = \frac{pb^2 a}{r} \cdot \frac{2}{3}$ , quæ explicat soliditatem totius elipsoidis.

856 Coroll. Cum sit circulus descriptus semiaxi conjugato tanquam radio  $= \frac{pb^2}{2r}$ , soliditas unius cylindri hujus basis, & altitudinis  $= 2a$ , axis major elipsis, erit  $= \frac{pb^2}{2r} \cdot 2a = \frac{pb^2 a}{r}$ ; ergo soliditas elipsoidis est ad eam cylindri qui circumscribi potest  $:: \frac{2}{3} : 1 :: 2 : 3$ ; & cum sit sphaera elipsoides axium conjugatorum æqualium, infertur quod etiam sphaera erit ad cylindrum qui & circumscribi potest  $:: 2 : 3$ ; hoc est, *elipsoides, & etiam sphaera est æqualis duobus tertiis cylindri circumscripti.*

Fig.

## ARTICULUS VIII.

## De Superficie curvarum solidorum revolutionis.

857 **T**heor. Formula generalis exprimens differentiale, aut elementum infinitesimum superficierum curvarum in solidis revolutionis est  $\frac{py}{r} \sqrt{dx^2 + dy^2}$ , representando  $y$  ordinatas,  $x$  abscissas, ex  $r : p$  rationem radii ad peripheriam.

105 Prout curva  $AM$  in sua revolutione describit super axim  $AQ$  superficiem curvam totius solidi, arcus infinitesimus  $Mm$  describet absdubio superficiem conii truncati, quæ erit differentiale, aut elementum infinitesimum superficierum solidi revolutionis, & ejus expressio erit productum ex  $Mm$  per circumferentiam circuli descripti radio ducto perpendiculariter è dimidio  $Mm$  super axim  $AQ$  (587), qui in nihilo differt à  $PM = y$  (801); sed  $Mm = \sqrt{dx^2 + dy^2}$  (845) &  $\frac{py}{r} =$  peripheriæ circuli, cujus radius  $= y$ : ergo reipsa illa superficies, aut elementum infinitesimum erit  $= \frac{py}{r} \sqrt{dx^2 + dy^2}$ .

858 Coroll. Cum sit  $\frac{py}{r} \sqrt{dx^2 + dy^2}$  formula generalis exprimens elementum infinitesimum superficierum curvarum in solidis revolutionis,  $S. \frac{py}{r} \sqrt{dx^2 + dy^2}$  exprimet totam superficiem curvam cujuslibet è illis solidis; sic ad applicandam formulam illam ad casus speciales, sumetur æquatio curvæ generatricis, deducetur valor ex  $y$  in expressionibus  $x$ , ac differentiando quantitatem resultantem, deducetur etiam valor ex  $dy^2$  in æquationi-



nibus  $x$ , &  $dx$ , quorum valores substituti in formula Fig. generali, ac integrando postmodum, dabunt superficiem quæsitam.

859 Prob. I. *Invenire superficiem curvam coni recti SBL.* 76

Sol. Cum hic formetur per revolutionem triang. rectanguli  $SLC$  circum cathetum  $CS = b$ , si supponimus  $CL = a$ ,  $cS = x$ , &  $cl = y$ , æquatio lineæ generatricis erit  $ax = by$  (707); ergo  $y = \frac{ax}{b}$  cujus æquatio differentiatâ dat  $dy = \frac{adx}{b}$   $dy^2 = \frac{a^2 dx^2}{b^2}$ ; substituendo hos valores ex  $y$ , & ex  $dy^2$  in formula generali elementis superficialis, habebimus  $\frac{py}{r} \sqrt{dx^2 + dy^2} = \frac{pax}{br} \sqrt{dx^2 + \frac{a^2 dx^2}{b^2}} = \dots$   
 $\frac{pax dx}{b^2 r} \sqrt{b^2 + a^2}$ ; ac consequenter  $S. \frac{py}{2} \sqrt{dx^2 + dy^2} =$   
 $S. \frac{pax dx}{b^2 r} \sqrt{b^2 + a^2} = \frac{pax^2}{2rb^2} \sqrt{b^2 + a^2}$ ; quæ expressio representat superficiem convexam cujuslibet portionis conicæ  $Sbl$ ; ac si supponimus  $x = b$  substituendo habebimus  $\frac{pax^2}{2rb^2} \sqrt{b^2 + a^2} = \frac{pa}{2x} \sqrt{b^2 + a^2}$ , expressio manifestans totam superficiem curvam coni; & quoniam  $\sqrt{b^2 + a^2}$  representat latus  $SL$  coni, &  $\frac{pa}{2r}$  semiperipheriam circuli suæ basis, sequitur quod superficies convexa unius coni æqualis est producto semiperipheriæ circuli suæ basis per suum latus  $SL$ .

860 Probl. 2. *Invenire superficiem spheræ cujus diameter = 2a.*

Sol. Æquatio curvæ generatricis, vel circuli est  $y^2 = 2ax - x^2$ ; ergo  $y = \sqrt{2ax - x^2}$ ,  $dy = \frac{adx - xdx}{\sqrt{2ax - x^2}}$   
d

Fig.  $dy^2 = \frac{a^2 dx^2 - 2ax dx^2 + x^2 dx^2}{2ax - x^2}$ , qui valores substituti in formula elementi superficialis dat  $\frac{py}{r} \sqrt{dx^2 + dy^2} = \frac{apdx}{r}$ ; ac consequenter  $S. \frac{py}{r} \sqrt{dx^2 + dy^2} = S. \frac{apdx}{r} = \frac{ap}{r} x$ , quæ expressio manifestat superficiem convexam fragmenti sphaerici cujuslibet; ac faciendo  $x = a$ , expressio praecedens convertetur in  $\frac{ap}{r} \cdot a$ , quæ repræsentabit superficiem convexam semisphaeræ, & ejus duplum  $\frac{ap}{r} \cdot 2a$  superficiem totius sphaeræ. Hoc significat, quod superficies curva cujuslibet portionis sphaericæ est semper æqualis producto unius circuli maximi sphaeræ per altitudinem correspondentem singulis illis portionibus.

## ARTICULUS IX.

### De Methodo inversa tangentium.

861 **H**ucusque, mediis æquationibus curvarum, cognitionem aperuimus earum proprietatum, subtang., tang., subnorm., norm., arearum, &c. Nunc, media harum proprietatum cujuslibet cognitione, investigabimus æquationem, & naturam curvæ cui correspondet; quod quidem nil est quam exequi, seu ad praxim reducere methodum inversam tangentium. Modus igitur huc aptandi calculum integralem, pendet ex circumstantiis, quas duo sequentes casus comprehendunt.

1.º Dum nobis dentur expressiones subtangentium, tangentium, ac cujuslibet è illis quantitibus, quæ medio calculo differentiali inveniuntur, eas æquabimus formulis generalibus quantitatum, quas repræsentant, hanc-

hancque æquationem postmodum integrando, resultabit Fig. æquatio curvæ, quæ desideratur.

2.<sup>o</sup> Si detur expressio alicujus arcus, areæ, solidi revolutionis, aut horum superficiei, differentiabitur illa expressio, cujus differentiale æquale fiet formulæ generali, dato respondententi, ac postea integrando illam æquationem, dum opus est, resultabit æquatio curvæ correspondentis.

862 Prob. 1. *Invenire curvam, cujus subtangens sit*  $= \frac{2y^2}{p}$ .

Sol. Fiat  $\frac{2y^2}{p} = \frac{ydx}{dy}$ , erit  $2ydy = pdx$ , quæ æquatio integrata dat  $y^2 = px$ , æquatio ad parabolam, cujus parameter  $= p$ .

863 Prob. 2. *Invenire curvam, cujus subtangens sit tertia proportionalis ad a — x, & ad y.*

Juxta conditionem erit  $a - x : y :: y : \frac{ydx}{dy}$ ; ergo  $adx - xdx = ydy$ , ac integrando  $ax - \frac{1}{2}x^2 = \frac{1}{2}y^2$ , vel  $y^2 = 2ax - x^2$ , æquatio ad circulum cujus diameter  $= 2a$ .

864 Prob. 3. *Invenire curvam, in qua subnormalis sit*  $= a$ .

Sol. Faciemus  $\frac{ydy}{dx} = a$ ; ergo  $ydy = adx$ , integrando erit  $\frac{y^2}{2} = ax$ , vel  $y^2 = 2ax$ , æquatio ad parabolam cujus parameter  $= 2a$ .

865 Prob. 4. *Invenire curvam, cujus area sit*  $= \frac{x^3}{3p}$ .

Sol. Differentiando hanc expressionem, & æquando differentiale ad formulam generalem elementi infinitesimi arearum

Fig.  $\frac{x^2 dx}{p} = y dx$ , vel  $x^2 = py$ , æquatio ad parabolam exter-  
nam, cujus parameter =  $p$ .

866 Prob. 5. *Invenire curvam generatricem solidi  
expressi per S. ( $pxdx - \frac{px^2 dx^2}{2r}$ ).*

Differentiando, & æquando cum formula generali so-  
lidi elementalis in solido revolutionis, erit  $pxdx - \frac{px^2 dx^2}{2r}$   
 $= \frac{py^2 dx^2}{2r}$ , vel  $2rpxdx - px^2 dx = py^2 dx$ ; ergo  $y^2 = 2xx$   
 $- x^2$ , quæ quidem æquatio manifestat, quod curva gene-  
ratrix est periphæria circuli, cujus diameter =  $2x$ , ac  
sphæra solidum propositum.

**F I N I S.**

- Pag. 5. *lin.* 28. illa , *leg.* alia : *ibid.* hæc , *leg.* & alia.
- Pag. 7. *lin.* 7. tertius , adde , dictus *productum* : *ibid. lin.* 28. nota &c. *leg.* multiplicator.
- Pag. 13. *lin.* 17. Tolle , *leg.* generatim : *ibid. lin.* 26. æque *leg.* in eodem respectu : *ibid. lin.* 29.  $\frac{1}{2} \frac{2}{4} \frac{4}{8}$ , *leg.*  $\frac{1}{2}$  ,  $\frac{2}{4}$  ,  $\frac{4}{8}$ .
- Pag. 15. *lin.* 2. nominatores , *leg.* numeratores.
- Pag. 18. *lin.* 1. sic , *leg.* sit.
- Pag. 20. *lin.* 4. centies , *leg.* centes : *ibid. lin.* 11. hora  $\frac{1}{24}$ , *leg.* hora  $1 = \frac{1}{24}$  : *ibid.* minutam  $\frac{1}{60}$  , *leg.* minutum  $1 = \frac{1}{60}$  : *ibid. lin.* 17. tolle , *leg.* immo duo vel tres.
- Pag. 29. *lin.* 29. illarum , *leg.* ejus.
- Pag. 30. *lin.* 2. mille , adde , nam : *ibid. lin.* 10. radice (§. 85) *leg.* radice (§. 85) & consequenter inveniri poterit summa cum facilitate : *ibid. lin.* 23. & residuum , *leg.* & subscribatur residuum : *ibid. lin.* 27. adjungatur : *per* , *leg.* adjungatur , atque *per*.
- Pag. 33. *lin.* 1. contineatur , *leg.* continetur : *ibid. lin.* 20. factoris , *leg.* numeratoris.
- Pag. 39. *lin.* 12.  $\overline{ef + qd.m}$  , *leg.*  $\overline{ef + qd} . m$  : *ibid.* brevius , *leg.* etiam.
- Pag. 42. *lin.* 24.  $b^3$  . *leg.*  $a^3$  . *ibid. lin.* ult. deinde , *leg.* hoc modo.
- Pag. 47. *lin.* 18. quantitas , *leg.* quantitatis.
- Pag. 51. *lin.* 10.  $\sqrt[mn]{d^{\frac{n}{n}}}$  , *leg.*  $\sqrt[mn]{c^{\frac{n}{d^n}}}$  : *ibid. lin.* 14. pro-

- ductum, *leg.* quotiens: *ibid.* *lin.* 20.  $\sqrt[n]{a}$ , *leg.*  $\sqrt[n]{a^n}$ .
- Pag. 52. *lin.* 3. radix  $n \sqrt[m]{a}$ , *leg.* radix  $n$  quantitatis  $\sqrt[m]{a}$ : *ibid.* *lin.* 6. radix.  $\frac{u}{s} \frac{a}{b} \sqrt[n]{\frac{c}{d}}$ , *leg.* radix  $\frac{u}{s}$  quantitatis  $\frac{a}{b} \sqrt[n]{\frac{c}{d}}$ .
- Pag. 57. *lin.* 22. exponens  $b$ , *leg.* exponens speciei  $b$ .
- Pag. 58. *lin.* 1.  $\pm \frac{b}{a}$ , *leg.*  $\frac{\pm b}{a}$ : *ibid.* *lin.* 20.  $32 a^5$ , *leg.*  $32 x^5$ .
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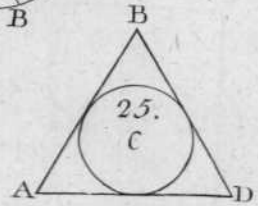
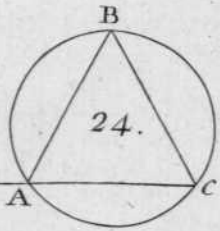
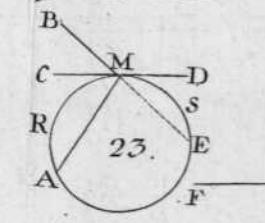
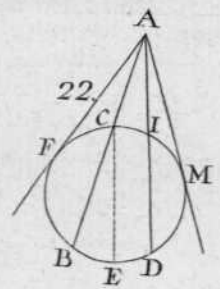
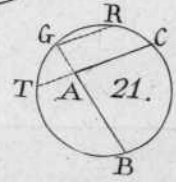
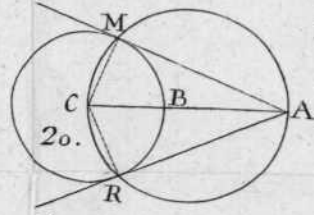
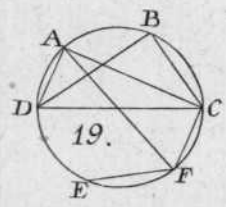
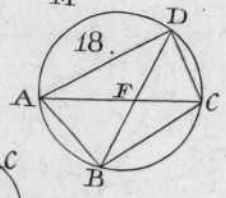
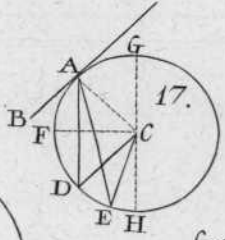
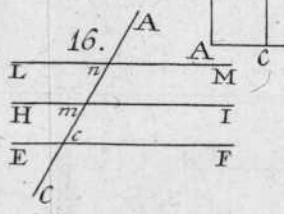
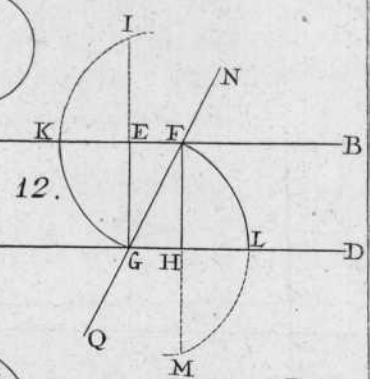
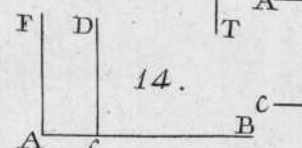
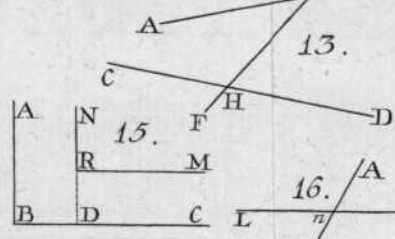
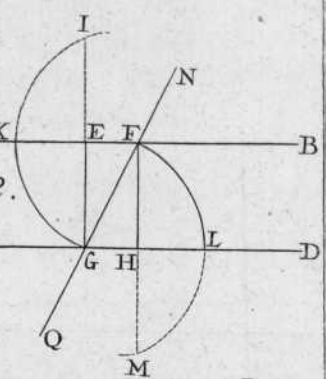
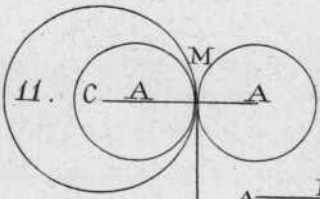
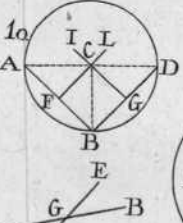
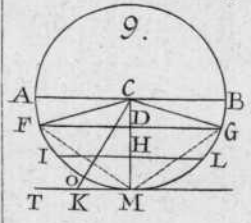
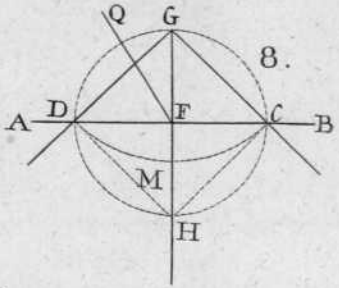
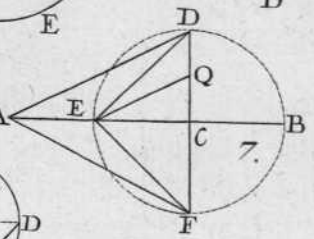
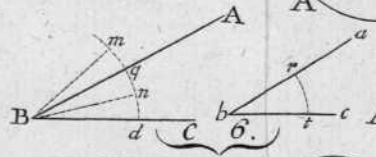
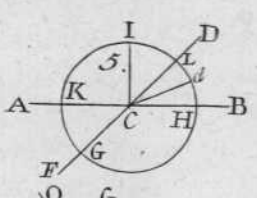
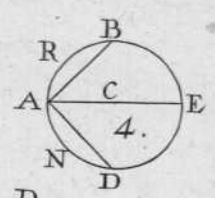
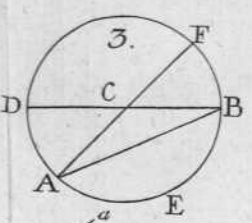
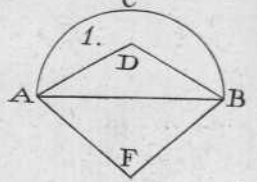
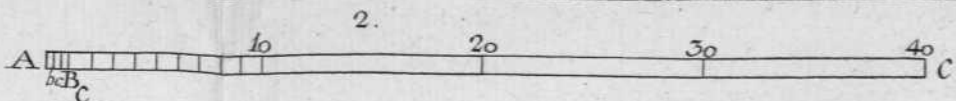
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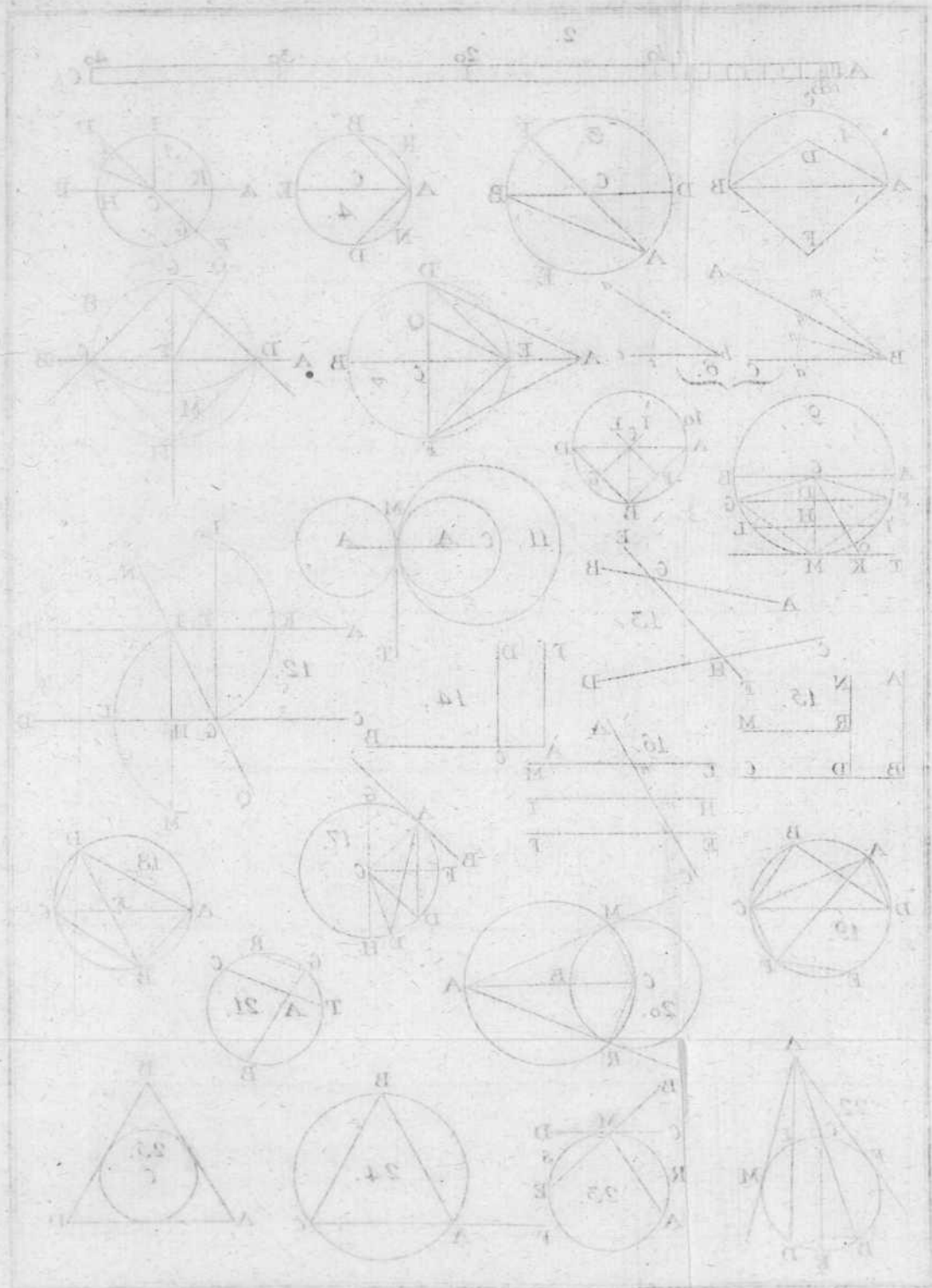
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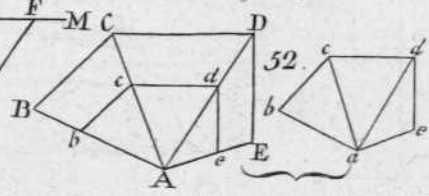
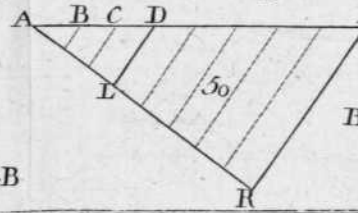
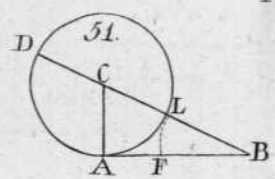
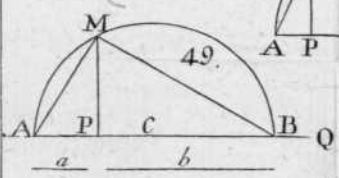
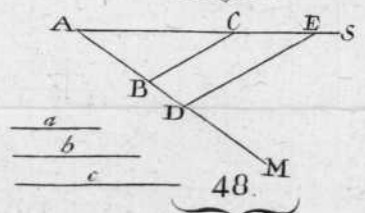
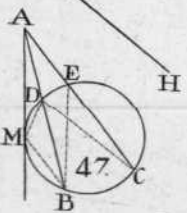
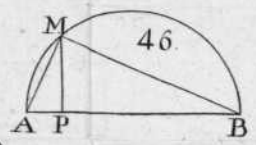
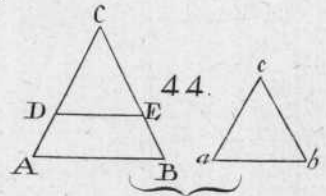
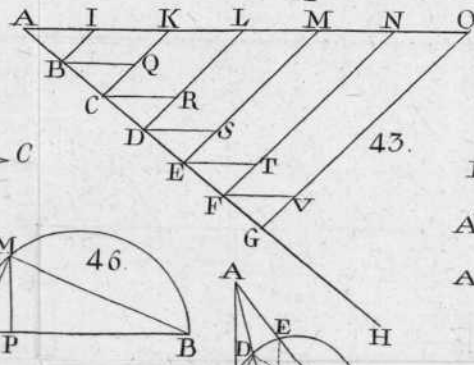
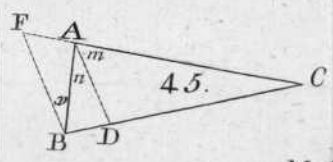
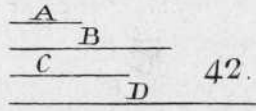
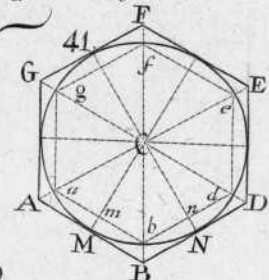
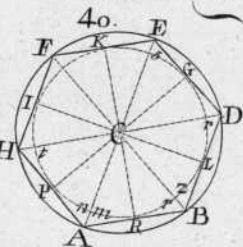
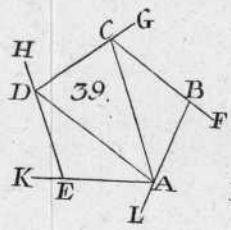
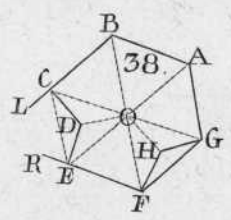
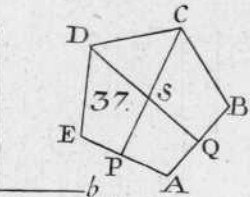
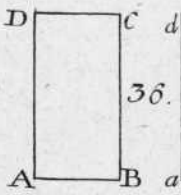
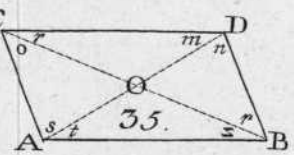
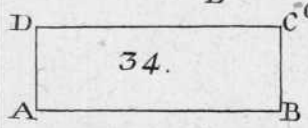
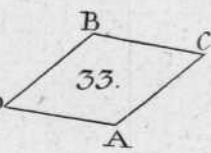
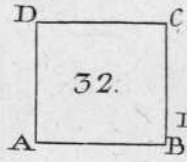
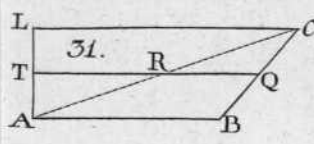
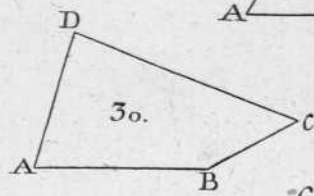
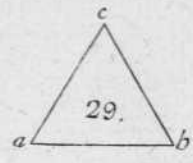
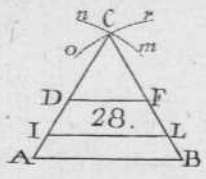
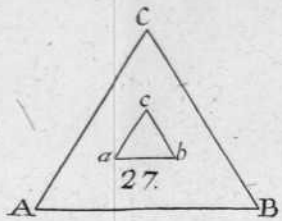
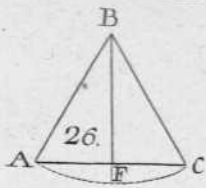
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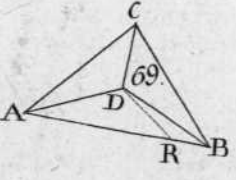
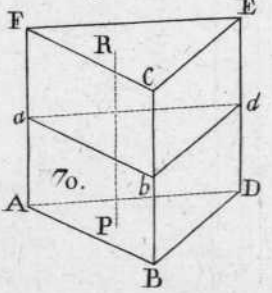
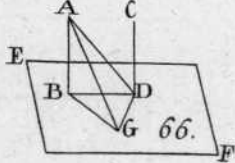
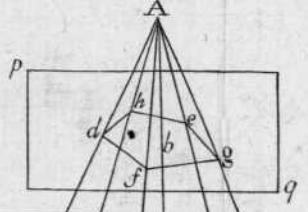
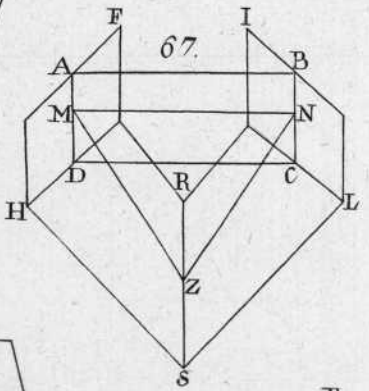
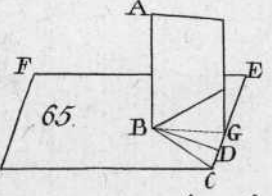
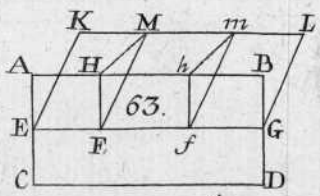
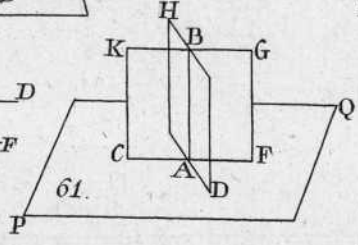
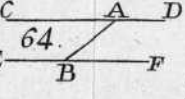
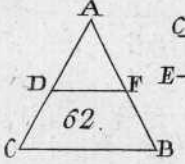
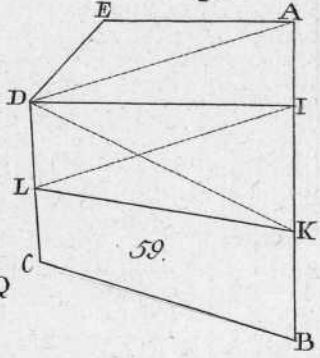
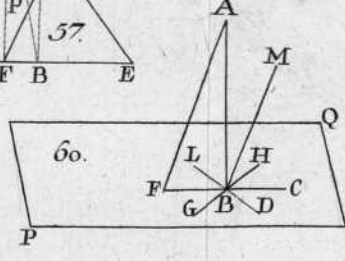
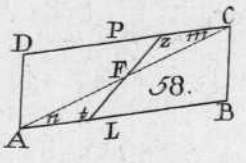
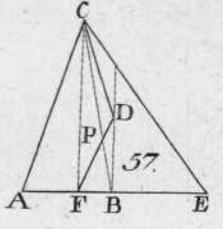
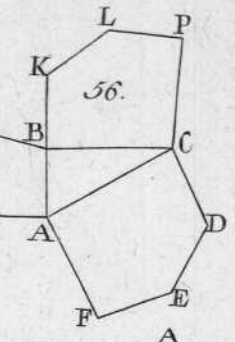
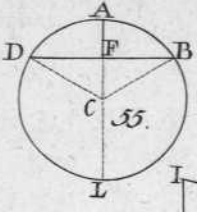
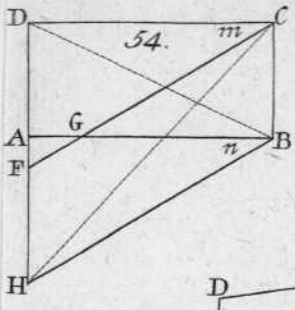
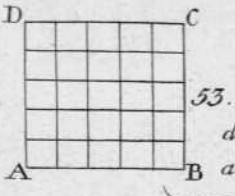
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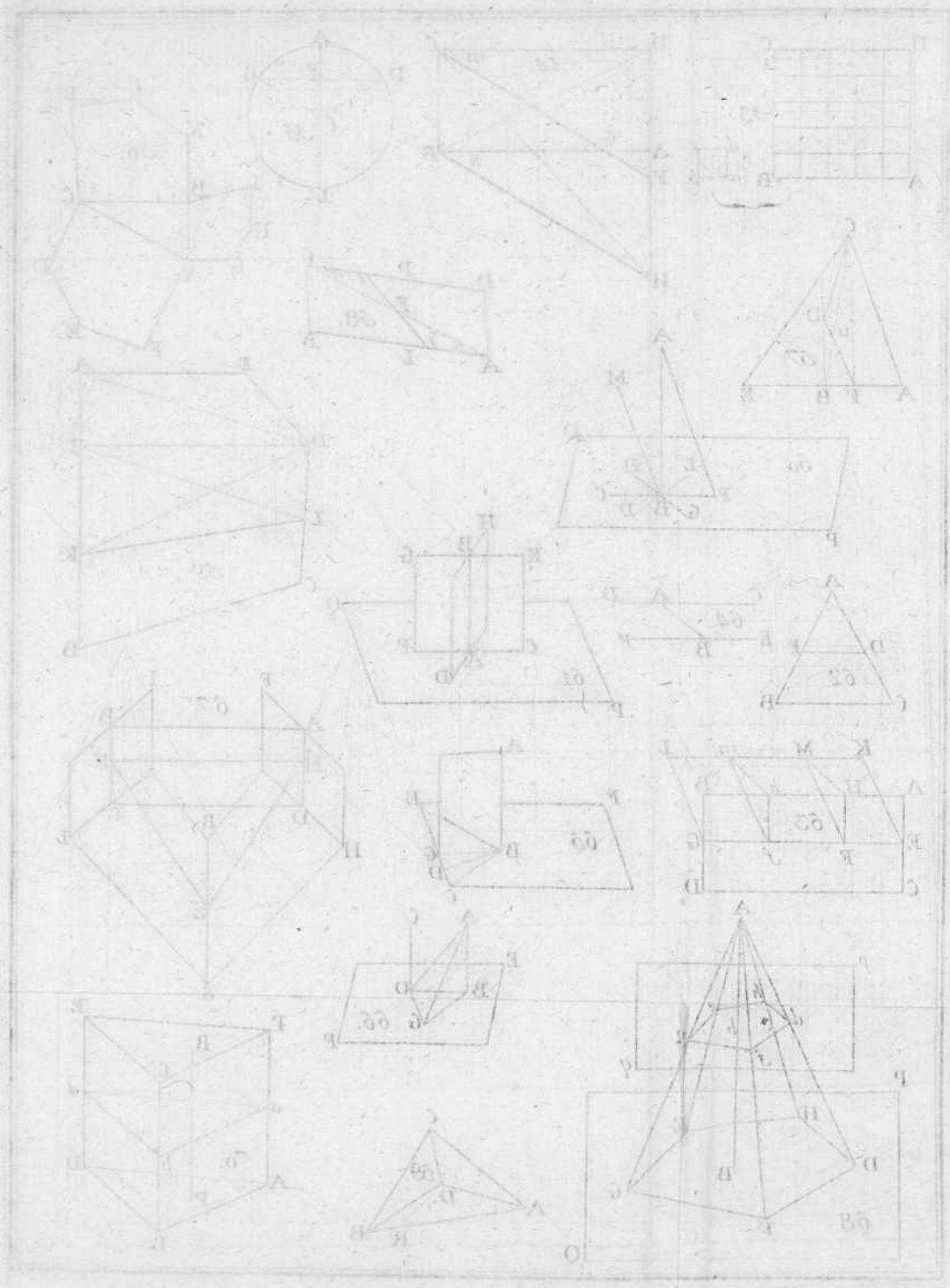




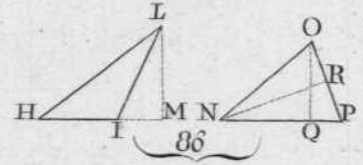
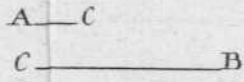
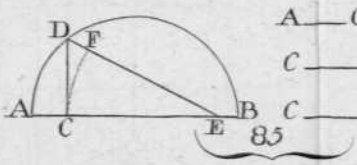
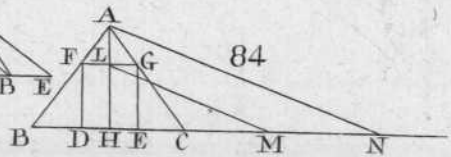
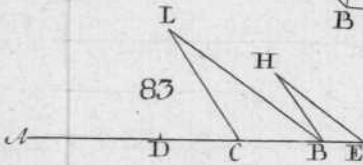
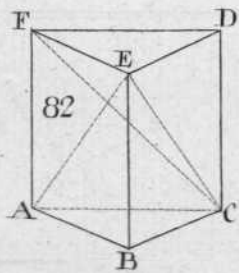
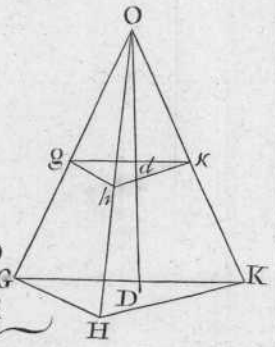
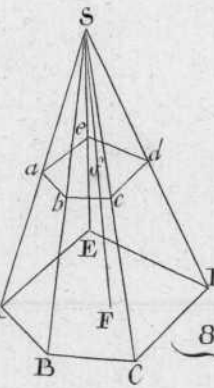
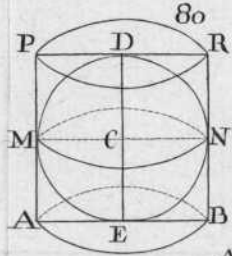
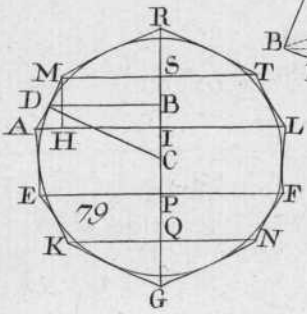
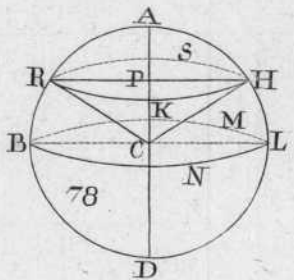
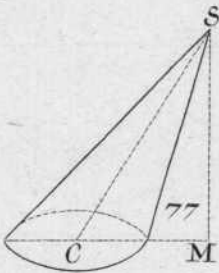
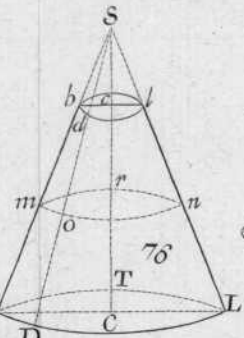
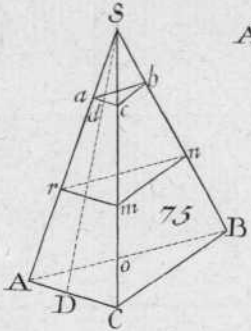
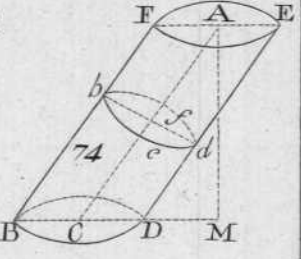
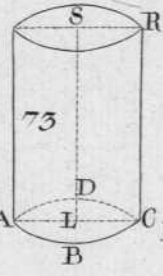
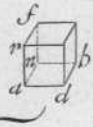
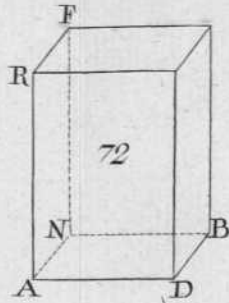
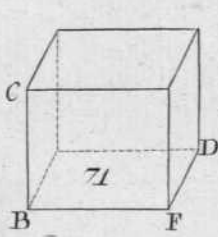


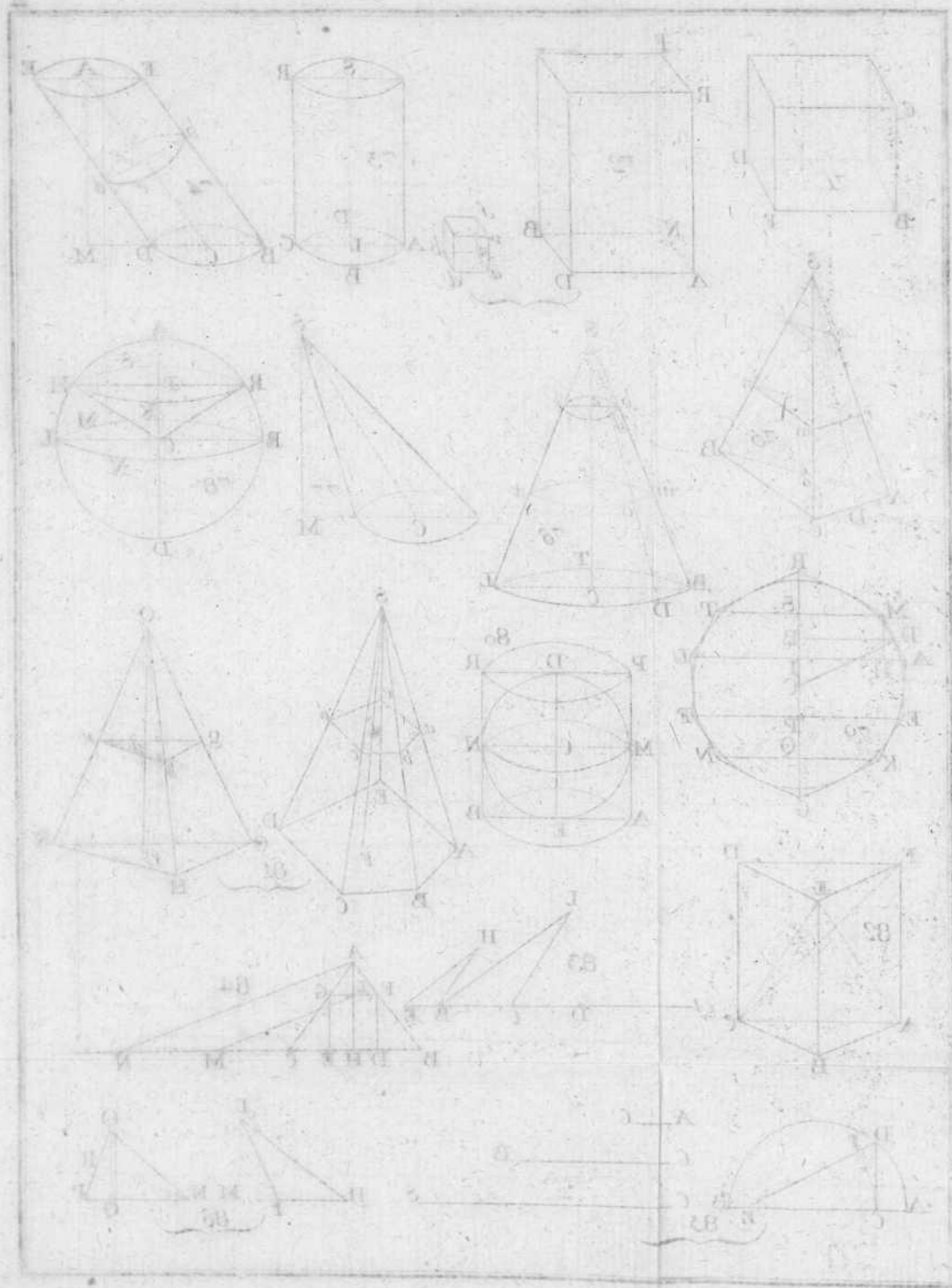


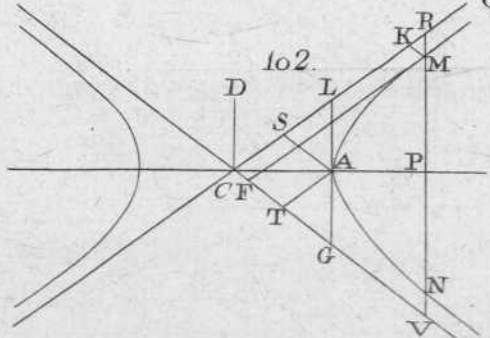
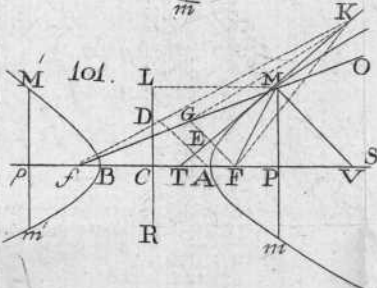
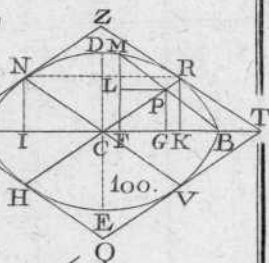
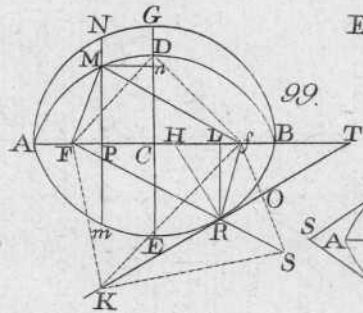
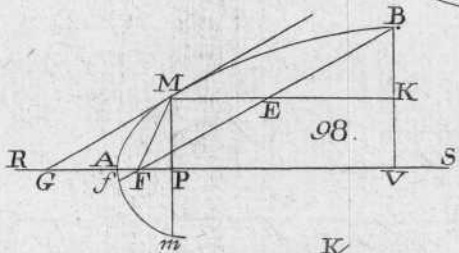
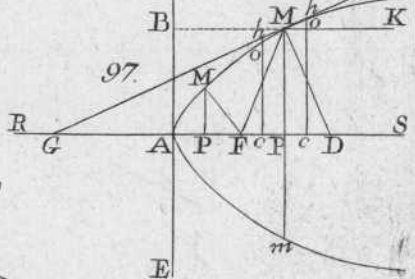
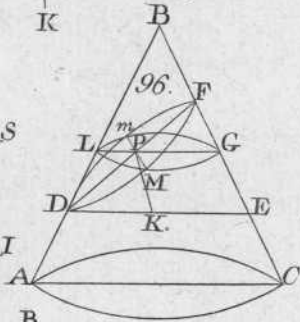
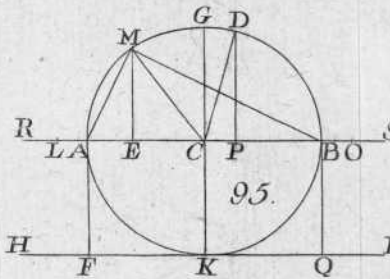
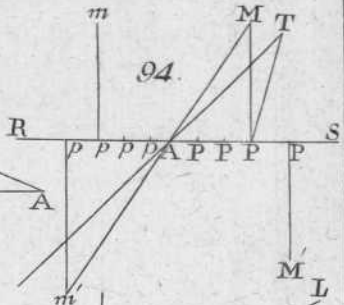
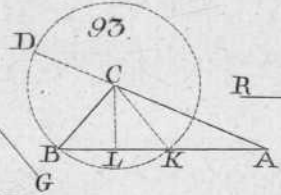
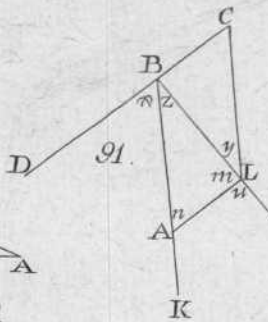
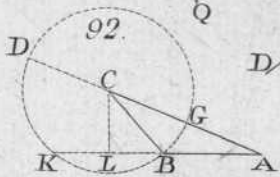
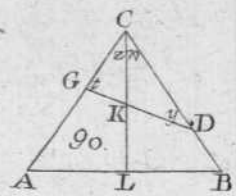
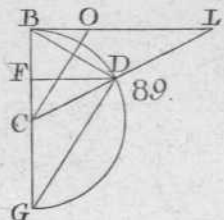
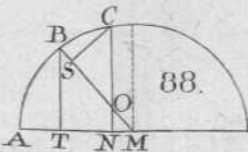
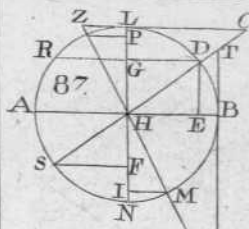


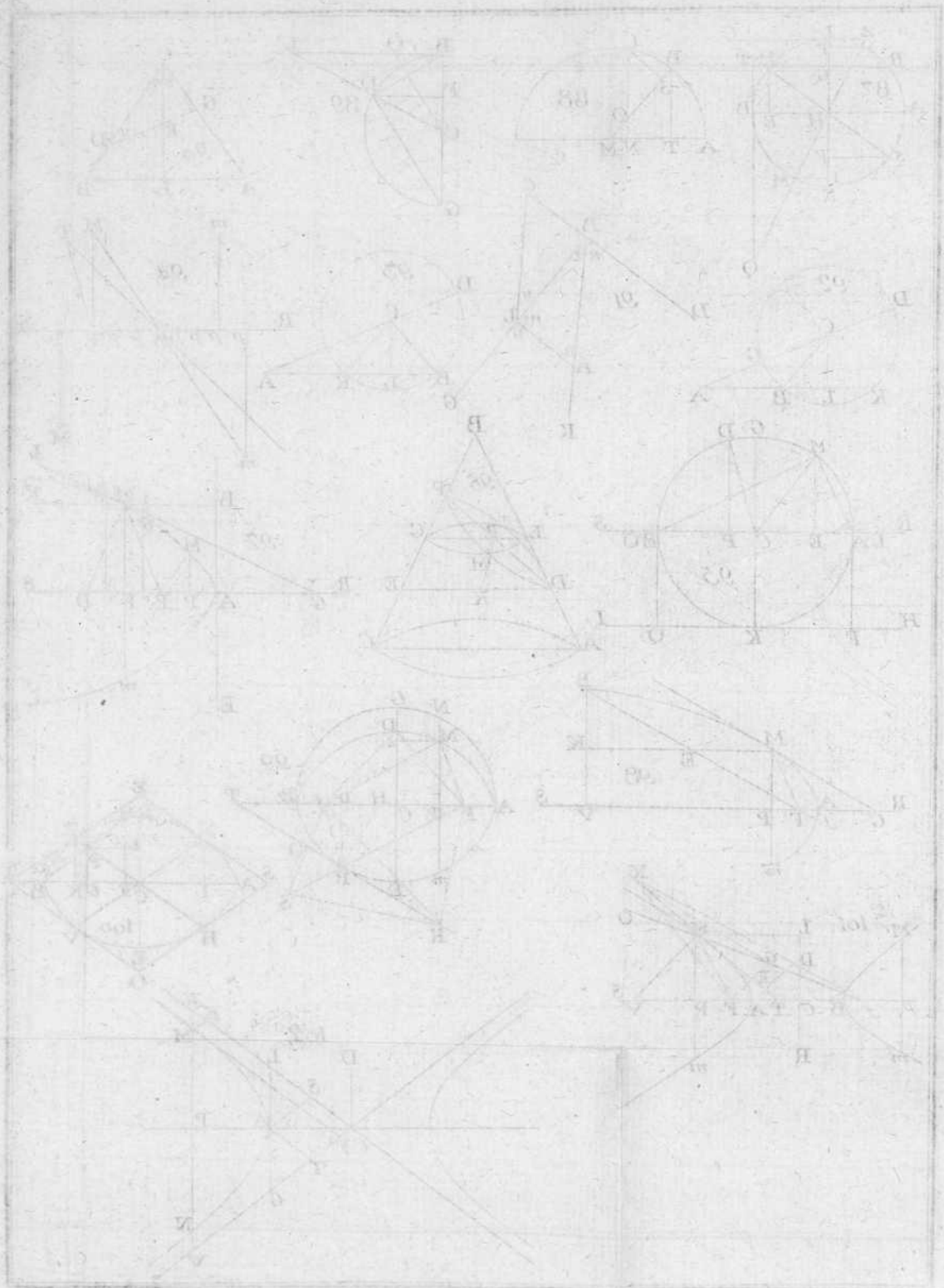


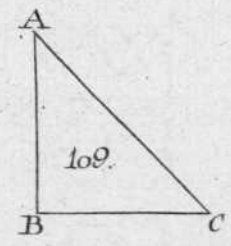
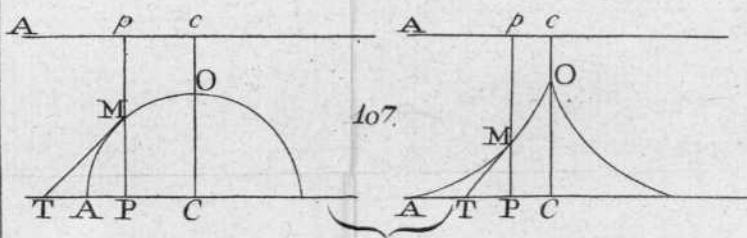
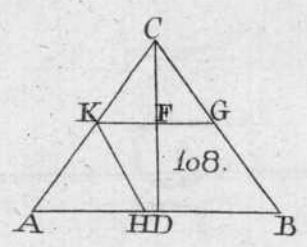
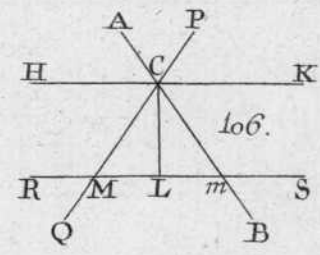
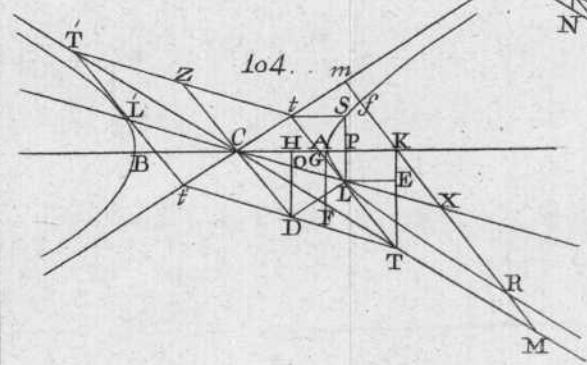
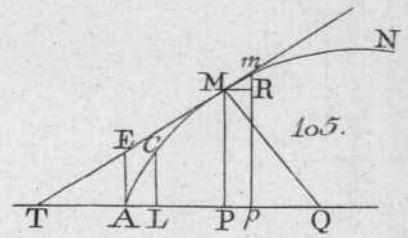
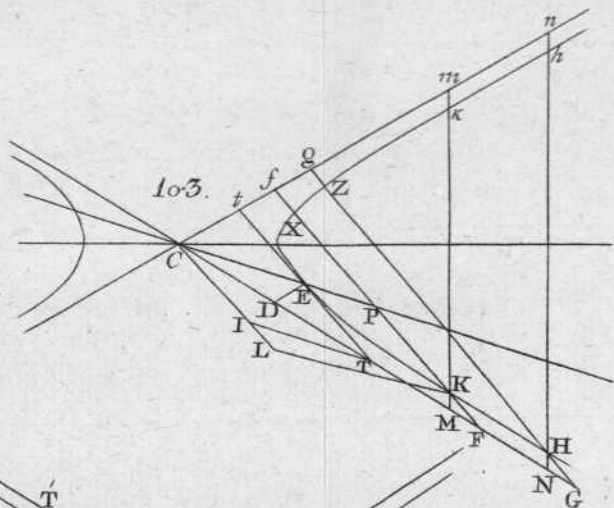


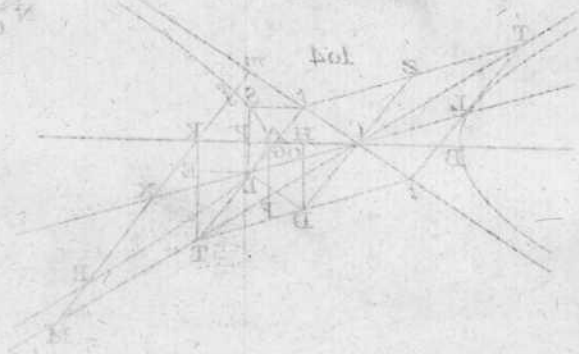
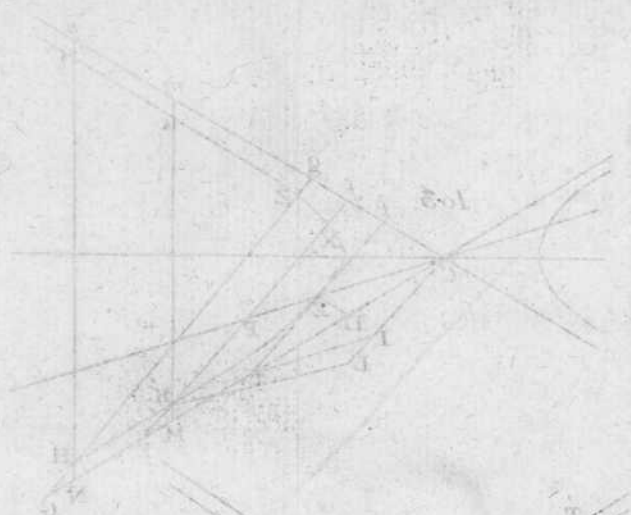
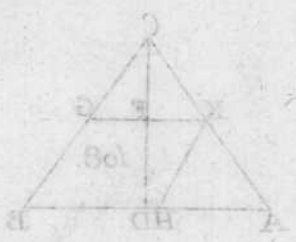
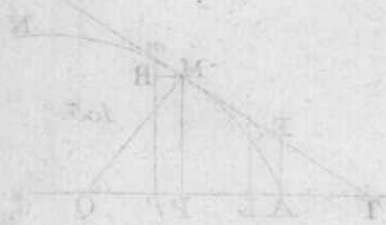


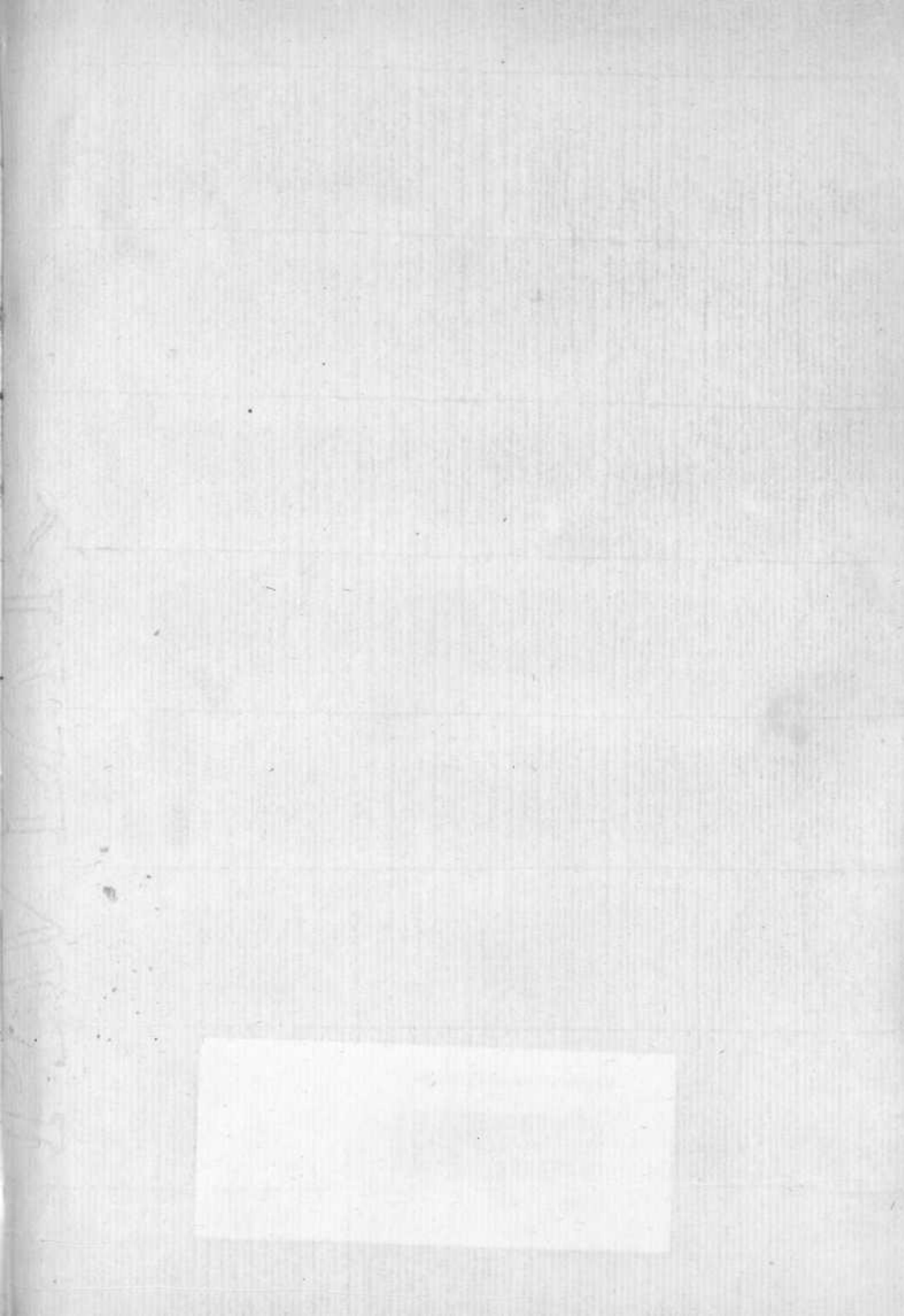










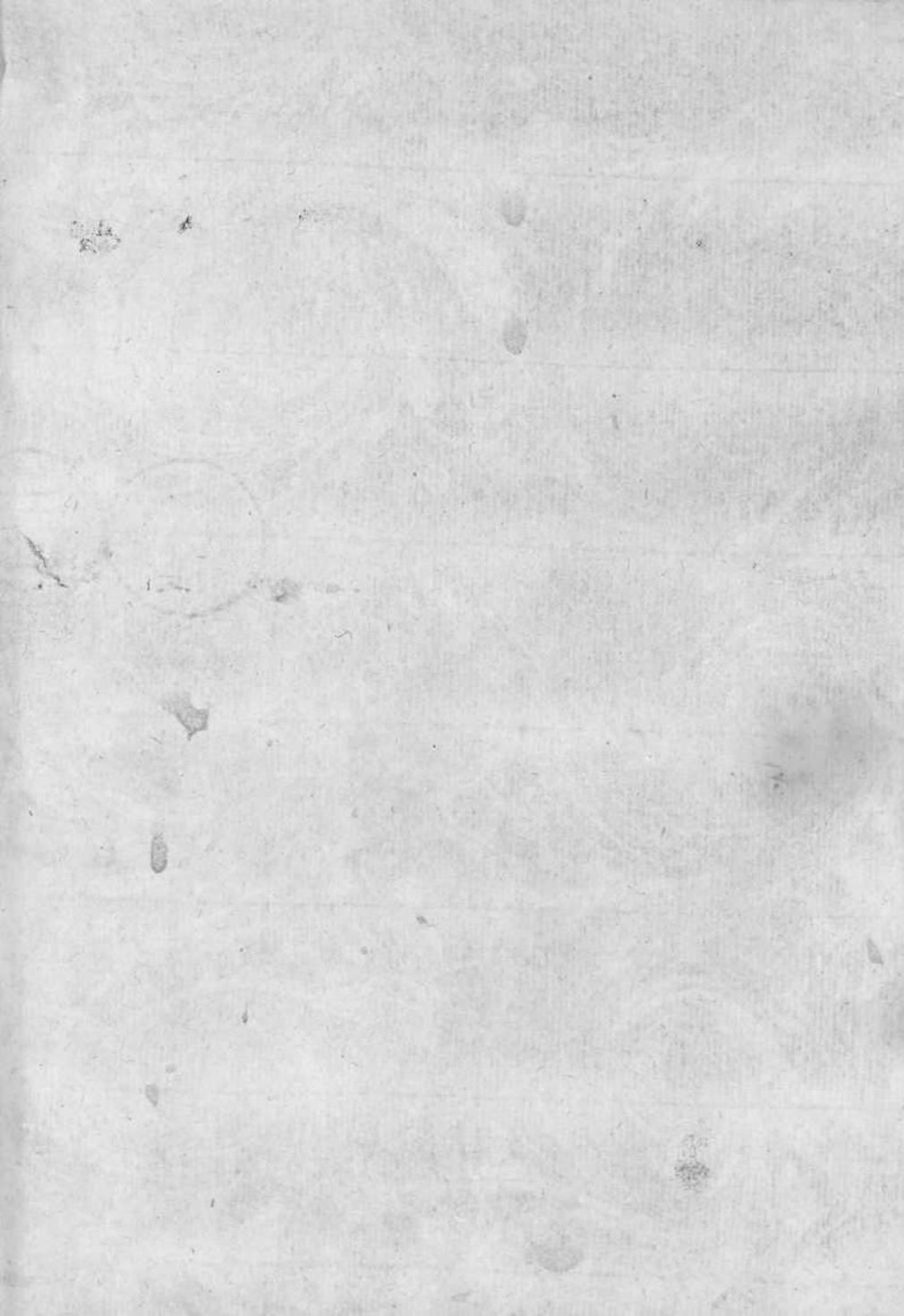


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